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Effects of leptonic CP violating phases in the left-right supersymmetric model

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Abstract. We analyze the effects of CP violating phases in a fully left-right extension of the minimal supersymmetric model. These phases appear from *both* the heavy and light neutrino sectors: two CKM-type phases, and four Majorana phases. We study observable effects of these phases in lepton flavor violating decays, such as the T-odd asymmetry in $\mu^+ \to e^+e^+e^-$, as well as in the leptonic electric dipole moments. We impose the experimental constraints from the mixing of light neutrinos and analyze cases in which the heavy and light neutrinos are either degenerate, or hierarchical, and highlight the dominant variables in each case. CP violating phases in both the heavy and light neutrino sectors of the left-right supersymmetric model have unique features which, if tested in the charged lepton sector, may distinguish the model from other supersymmetric scenarios.

1 Introduction

The evidence for neutrino oscillations in both solar [1] and atmopsheric [2] neutrino measurements has provided the first experimental confirmation of physics beyond the Standard Model (SM). In particular, it has given a boost to the study of leptonic phenomenology as a forefront of new physics signals. The most commonly accepted explanation for small neutrino masses is provided by the see-saw mechanism [3], in which large Majorana masses for the right-handed neutrinos induce small masses for the light neutrinos. If the neutrinos are massive and mixed, lepton Yukawa interactions are no longer flavor diagonal, and there exists a source of leptonic flavor and CP violation, analogous to the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in the quark sector. In the simplest extension of the SM, three heavy singlet neutrinos are needed. If one chooses a model in which quadratically-divergent contributions to the Higgs mass introduced by the heavy neutrinos are cancelled, then one is led to the minimal supersymmetric standard model (MSSM) with singlet right-handed neutrinos [MSSM(RN)] [4].

However MSSM(RN) introduces right-handed singlet neutrinos in a rather *ad-hoc* manner, through terms in the Lagrangian, not dictated by any symmetries in the model. Some Supersymmetric Grand Unified Theories (SUSY GUT-s) alleviate such problems, although again, in SU(5), right-handed neutrinos must be added to the existing theory. In this paper we examine the simplest model which accomodates naturally the right-handed neutrinos, the Left-Right Supersymmetric Model (LRSUSY) [5–8]. Left-right supersymmetry extends the MSSM gauge group to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which would then break spontaneously to the group $SU(2)_L \times U(1)_Y$ [5]. Originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses [7], the LRSUSY model can be embedded in a supersymmetric grand unified theory such as $SO(10)$ [9]. Additional support for left-right theories is provided by building realistic brane worlds from Type I strings [10].

In this paper we show that LRSUSY has another feature: it provides new sources of leptonic CP violation through the flavor structure of the right-handed doublets. Because the model is left-right symmetric, there exists a CKM matrix in the right-handed lepton sector. In addition to a CKM-type CP violating phase, the right-handed neutrino mixing contains Majorana phases. Although the right-handed neutrinos are heavy, virtual effects of the massive neutrinos affect the renormalization group equations (RGE) of the slepton mass and trilinear coupling matrices, by providing extra contributions to the off-diagonal terms which induce lepton flavor violation (LFV). One could study the direct consequences of such phases in neutrino oscillations, but there comparison with experiment is yet premature. Or, one could study the effects of these phases in charged lepton phenomenology, the advantage there being that precise measurements exist, and several bounds will be improved significantly in the future. The current experimental limits (with future sensitivities in brackets) of lepton-flavor violating experiments involving charged leptons are given below:

$$\begin{aligned} B.R.(\mu \to e\gamma) &< 1.2 \cdot 10^{-11}(10^{-14}) \quad [11,12] \\ R(\mu^- Ti \to e^- Ti) &< 6.1 \cdot 10^{-13}(10^{-14}) \quad [13,14] \\ B.R.(\tau \to \mu\gamma) &< 1.1 \cdot 10^{-6}(10^{-9}) \quad [15,16] \\ B.R.(\tau \to e\gamma) &< 2.7 \cdot 10^{-6} \quad [17] \\ B.R.(\mu^+ \to e^+e^+e^-) &< 1.0 \cdot 10^{-12}(10^{-16}) \quad [18,19] \end{aligned} \tag{1}$$

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The last process has such a high sensitivity that it offers the possibility to measure the T-odd, CP-violating asymmetry, $A_T(\mu^+ \to e^+e^+e^-)$.

Another area to test for CP violation would be the electric dipole moment of the electron or the muon. At present, the electric dipole moment of the electron is measured with high sensitivity [17]:

$$d_e < 4.3 \cdot 10^{-27} e\ cm \tag{2}$$

In LRSUSY the electric dipole moments will be shown to provide information about the mass structure in the neutrino sectors, rather than the CP violating phases.

A study of the effects of CP phases in this model is interesting from two points of view: on one hand, LRSUSY provides a different manifestation of leptonic CP violation from the MSSM(RN). On the other hand, LFV decays and electric dipole moments provide a test of the CP violating phases in the heavy neutrino sector through direct dependence of the low energy observables on the phases in the right-handed sector.

We investigate LFV decays and leptonic electric dipole moments as a source of information on the mass structure and CP phases in the light and heavy neutrino sectors. We discuss the parametrization of the leptonic Yukawa couplings, including the effects of CP violating phases and of the renormalization group equations running of soft-breaking terms. We assume universal boundary conditions at the GUT scale, thus avoiding large flavor violations *ab initio*; LFV will be introduced by radiative corrections only. We analyze the dependence of the T-odd asymmetry in $B.R.(\mu^+ \to e^+e^+e^-)$ on the Majorana and CKM-types phases in the light and heavy neutrino sectors. We show that, as in MSSM(RN), the T-odd asymmetry information is complimentary to the branching ratio for $\mu \to e\gamma$, but the dependence on the angles is very different from MSSM(RN). We pay particular attention to the heavy neutrino CP-violating oscillation phase σ and the corresponding two Majorana phases ψ_2 and ψ_3. We show the results of the analysis in distinct scenarios, assuming the possibility that either the light or the heavy neutrinos may be hierarchical or (quasi)degenerate, and how, in the first case, heavy neutrino mass ratio information can be obtained from lepton-flavor conserving processes.

The paper is organized as follows: we review the LRSUSY model and its sources of flavor and CP violation in Sect. 2. In Sect. 3 we describe the scenarios for light and heavy neutrino mixings considered. In Sect. 4 we discuss the dominant contributions to the T-odd asymmetry in $B.R.(\mu^+ \to e^+e^+e^-)$ and the electron EDM in the presence of slepton mixing. Our numerical analysis is included in Sect. 5, and we conclude in Sect. 6.

2 Sources of leptonic CP violation in the left-right supersymmetric model

The LRSUSY symmetry group,

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L},$$

contains three generations of quark and lepton chiral superfields [7]. The Higgs sector consists of two Higgs bidoublets, $\Phi_1(\frac{1}{2}, \frac{1}{2}, 0)$ and $\Phi_2(\frac{1}{2}, \frac{1}{2}, 0)$, which are required to give non-vanishing Cabibbo-Kobayashi-Maskawa quark mixing. In addition, Higgs triplet fields $\Delta_L(1, 0, 2)$ and $\Delta_R(0, 1, 2)$, which transform as the adjoint representation of $SU(2)_L$ and $SU(2)_R$, respectively, are introduced to provide spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. Triplets rather than doublets are preferred because a large Majorana mass can be generated (through the seesaw mechanism) for the right-handed neutrino and a small one for the left-handed neutrino [6]. The number of triplets must be doubled, and new triplets $\delta_L(1, 0, -2)$ and $\delta_R(0, 1, -2)$, with quantum number $B-L = -2$, are introduced for providing anomaly cancellation in the fermionic sector. The superpotential of the LRSUSY model is:

$$\begin{aligned} W = & \mathbf{Y}_Q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + \mathbf{Y}_L^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c \\ & + i\mathbf{Y}_{LR}(L^T \tau_2 \Delta_L L + L^{cT} \tau_2 \Delta_R L^c) \\ & + M_{LR}\left[Tr(\Delta_L \delta_L + \Delta_R \delta_R)\right] \\ & + \mu_{ij} Tr(\tau_2 \Phi_i^T \tau_2 \Phi_j) + W_{NR} \end{aligned} \tag{3}$$

with W_{NR} possible non-renormalizable terms arising from higher scale physics or Planck scale effects [20]. The presence of these terms is important when the SUSY breaking scale is above M_{W_R}, insuring the existence of an R-parity conserving ground state [21]. The M_{W_R} scale can be either relatively low (around 10 TeV) or much higher ($M_{W_R} \approx 10^{12} - 10^{14}$ GeV). Each possibility is achieved through a different mechanism of $SU(2)_R$ breaking [5], and neither is ruled out experimentally. We chose here a large M_{W_R} scale since it is favored by the seesaw mechanism.

Neutral Higgs fields acquire non-zero vacuum expectation values ($VEV's$) through spontaneous symmetry breaking of both parity and $SU(2)_R$:

$$\langle \Delta \rangle_L = 0, \quad \langle \Delta \rangle_R = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \text{ and}$$

$$\langle \Phi \rangle_{1,2} = \begin{pmatrix} \kappa_{1,2} & 0 \\ 0 & \kappa'_{1,2} e^{i\omega} \end{pmatrix}.$$

During the first stage of breaking, the right-handed gauge bosons get masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons, which acquire masses proportional to κ_1 and κ_2 at the second stage of breaking. In equation (3), $\mathbf{Y}_Q$ and $\mathbf{Y}_L$ are the Yukawa couplings for the quarks and leptons with bidoublet Higgs bosons, respectively, and $\mathbf{Y}_{LR}$ is the coupling for the leptons and triplet Higgs bosons. LR symmetry requires all $\mathbf{Y}$-matrices to be Hermitean in the generation space and $\mathbf{Y}_{LR}$ matrix to be symmetric: $\mathbf{Y}^i_{Q,L} = \mathbf{Y}^{i\dagger}_{Q,L}$, $\mathbf{Y}_{LR} = \mathbf{Y}^t_{LR}$. The Yukawa matrices cause flavor violation through misalignment between the particle and sparticle bases. In addition, soft supersymmetry breaking masses for the charged slepton fields also induce LFV. After symmetry breaking, the neutrino mass Lagrangian is:

$$-2\mathcal{L}_{\rm mass} = \bar{n}_L^c M_\nu n_R + \bar{n}_R^c M_\nu^* n_L^c \tag{4}$$

There are 6 weak neutrino eigenstates which form:

$$N_R = \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} \quad \nu_R^c = C\bar{\nu}_L^T, \tag{5}$$

and the symmetric 6×6 mass matrix M_ν is given by:

$$M_{\nu_\alpha} = \begin{pmatrix} 0 & m_\alpha^D \\ m_\alpha^{D\dagger} & M_N \end{pmatrix} \tag{6}$$

with $m_\alpha^D = Y_L^1 \kappa_1 + Y_L^2 \kappa_2'$, and $M_N = 2Y_{LR} v_R$. The mass of the α-flavor neutrino is given by the canonical seesaw formula:

$$m_{\nu_\alpha} = -\frac{\left(m_\alpha^D\right)^2}{M_N} \tag{7}$$

To find the neutrino eigenstates, we perform a unitary transformation: $n_R = U\hat{n}_R$, such that $U^T M_\nu U = \hat{M}_\nu$ is a diagonal, positive mass matrix, and

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\bar{\hat{n}}_L^c \hat{M}_\nu \hat{n}_R + h.c \equiv -\frac{1}{2}\bar{N}\hat{M}_\nu N, \tag{8}$$

with $N = \hat{n}_R + \hat{n}_L^c \equiv \hat{n}_R + C\bar{\hat{n}}_R^T$, and

$$U = \begin{pmatrix} U_L^* \\ U_R \end{pmatrix} \tag{9}$$

where

$$\nu_L = U_L \hat{\nu}_L^c \equiv U_L(P_L N), \text{ and } \nu_R = U_R \hat{\nu}_R \equiv U_R(P_R N).$$

Similarly, the charged lepton mass matrix is:

$$M_l = Y_L^1 \kappa_1' + Y_L^2 \kappa_2 \tag{10}$$

and is diagonalized by the unitary transformation: $U_L^{l\dagger} M_l U_R^l = \hat{M}_l$, with $\hat{M}_l$ a diagonal, positive 3×3 mass matrix. If we denote the physical lepton fields by $l_{L,R} = U_{L,R}^l \hat{l}_{L,R}$, we can define the 6×3 leptonic charged current interaction matrices (the leptonic Cabibbo-Kobayashi-Maskawa matrices) as:

$$K_L^{CKM\ \dagger} = U_L^\dagger U_L^l \tag{11}$$
$$K_R^{CKM\ \dagger} = U_R^\dagger U_R^l \tag{12}$$

Since U_L^l and U_R^l are diagonal matrices, the mass matrix for the light neutrino states M_ν can then be diagonalized by the unitary matrix U_L,:

$$U_L^T M_\nu U_L = M_\nu^D \tag{13}$$

where M_ν^D =diag$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$. Given the fact that neutrinos are Majorana particles, the U_L matrix can be expressed as:

$$U_L = K_L^{CKM}(\theta, \delta) P(\phi) \tag{14}$$

where $P(\phi)$ =diag$(e^{-i\phi_1}, e^{-i\phi_2}, 1)$ and:

$$K_L^{CKM}(\theta,\delta) \tag{15}$$
$$= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

with $c(s)_{ij} = \cos(\sin)\theta_{ij}$. The mixing angles θ_{ij} and the CP-violating phase δ are measurable in neutrino oscillation experiments.

We perform a similar analysis for the heavy neutrino sector. The matrix for the heavy neutrino states is diagonalizable by the unitary matrix U_R,

$$U_R^T M_N U_R = M_N^D \tag{16}$$

where M_N^D =diag$(M_{N_1}, M_{N_2}, M_{N_3})$. The matrix U_R matrix can be expressed as:

$$U_R = K_R^{CKM}(\beta,\sigma) P(\psi) \tag{17}$$

where $P(\psi)$ =diag$(1, e^{-i\psi_2}, e^{-i\psi_3})$ and $K_R^{CKM}(\beta,\sigma)$ has the same form as the CKM matrix in the left-hand sector, but is a function of the independent (unknown) angles β and σ. We proceed to show that in LRSUSY these angles enter the low energy LFV processes through the RGE of soft-symmetry breaking terms in the Lagrangian.

The SUSY-breaking terms for the Higgs bosons and lepton sector in LRSUSY is given by:

$$\begin{aligned}\mathcal{L}_{soft} = &\Big[\mathbf{A}_L^i \mathbf{Y}_L^{(i)} \tilde{L}^T \tau_2 \Phi_i \tau_2 \tilde{L}^c \\ &+\mathbf{A}_{LR}\mathbf{Y}_{LR}(\tilde{L}^T \tau_2 \Delta_L \tilde{L} + L^{cT}\tau_2 \Delta_R \tilde{L}^c) \\ &+m_\Phi^{(ij)2}\Phi_i^\dagger \Phi_j\Big] \\ &+\Big[(m_L^2)_{ij}\tilde{l}_{Li}^\dagger \tilde{l}_{Lj} + (m_R^2)_{ij}\tilde{l}_{Ri}^\dagger \tilde{l}_{Rj}\Big] \\ &-M_{LR}^2\left[Tr(\Delta_R\delta_R) + Tr(\Delta_L\delta_L) + h.c.\right] \\ &-[B\mu_{ij}\Phi_i\Phi_j + h.c.]\end{aligned} \tag{18}$$

where $\mathbf{A}_L$, $\mathbf{A}_{LR}$ are soft-parameter matrices which provide additional sources of flavor violation. Inter-generational and left-right slepton mixing are responsible for the off-diagonal nature of the matrices, and for flavor violation. Explicit expressions for the slepton mass matrices in LRSUSY have appeared previously [22].

Denoting the charged slepton mixing matrix by $V_{L,R}$, we express the slepton mixing as:

$$\tilde{l}_{\alpha L,R} = (V_{L,R})_{\alpha i}\tilde{l}_i^{L,R} \tag{19}$$

with $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$. The left-left and right-right slepton mixings are approximately block-diagonal, while the left-right mixings are proportional to values of the trilinear parameters A_L. At energies below M_{GUT}, radiative corrections generate the off-diagonal entries in the slepton mass matrix via the RGE. These corrections are parametrized in LRSUSY using the logarithmic approximation as [23]:

$$(\delta\tilde{m}_L^2)_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2 + a_0^2)(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij}$$

$$
\begin{aligned}
&\times \log \frac{M_{GUT}}{M_N} \\
(\delta \tilde{m}_R^2)_{ij} &= (\delta \tilde{m}_L^2)_{ij} \\
(\delta A_L)_{ij} &\approx -\frac{A_0}{8\pi^2} \\
&\times \left[\left(3Y_L + \frac{1}{2} Y_{LR} \right) Y_L^\dagger Y_L + 2 Y_L Y_{LR}^\dagger Y_{LR} \right]_{ij} \\
&\times \log \frac{M_{GUT}}{M_N} \\
(\delta A_{LR})_{ij} &\approx -\frac{3A_0}{8\pi^2} [Y_{LR}(2Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})]_{ij} \\
&\times \log \frac{M_{GUT}}{M_N} \qquad (20)
\end{aligned}
$$

where the second equation is a consequence of left-right symmetry. Note that in the leading-logarithmic approximation both Yukawa couplings from the light (Y_L) and heavy (Y_{LR}) sectors enter the equation for the slepton mass matrices. The LFV interactions induced by slepton observables will depend on the phases in both the heavy and light neutrino sector, even if the heavy neutrinos decouple from the low-energy spectrum. This is a residual LRSUSY effect at low energies. The feature present in MSSM(RN), in which CP-violating phases are induced only in the off-diagonal elements of the slepton mass matrix, persists here.

Expressing the combination of Yukawa couplings appearing in the slepton mass matrix, we obtain, using the Casas and Ibarra parametrization [4]:

$$
\begin{aligned}
(Y_L)_{ki} &= \frac{1}{v \sin\beta} \mathrm{diag}(\sqrt{M_{N_1}}, \sqrt{M_{N_2}}, \sqrt{M_{N_3}}) R_{kl} \\
&\times \mathrm{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}) (U_L^\dagger)_{li} \\
(Y_{LR})_{ki} &= \frac{1}{2 v_R} (U_R)^*_{kl} \\
&\times \mathrm{diag}(M_{N_1}, M_{N_2}, M_{N_3}) (U_R^\dagger)_{li} \qquad (21)
\end{aligned}
$$

with R_{kl} an auxiliary complex orthogonal matrix. Before proceeding to find explicit expressions for the Yukawa couplings as functions of neutrino masses and mixings, we comment on the role of sneutrino masses in affecting LFV decays and CP violating parameters. The sneutrino mass matrix is expressed by a 12×12 matrix, but an effective 6×6 matrix for the light sneutrinos using the seesaw mechanism [24] can be obtained. The seesaw mechanism in the sfermion sector insures small mixing between the right-handed and the left-handed sneutrinos. The left-right elements of the sneutrino mass matrix are proportional to the Dirac neutrino mass and can be significant, while the right-right element of the sneutrino mass matrix is very heavy. This suppresses the mixing of sneutrinos by $1/M_N^2$, with M_N the right-handed neutrino mass. Thus in the light sneutrino sector the Dirac terms mix almost degenerate states and do not induce considerable mixing [25]. Flavor violating decays are therefore dominated by the charged slepton mixing.

3 Light neutrino mixing and parameters in the leptonic sector

In the leptonic sector, LRSUSY has three light neutrino mixing angles θ_{ij}, three CP-violating light neutrino mixing angles: δ and $\phi_{1,2}$; and similarly, for the heavy neutrino sector, the mixing angles β_{ij} and the CP violating angles σ and $\psi_{2,3}$. In addition to these, there are three charged lepton, three light neutrino and three heavy neutrino mass parameters. Thus, a complete analysis eludes us. We will however consider the effects of the masses and mixings in the neutrino sector in some interesting scenarios which were previously singled out in the literature. We also take advantage of relationships established between the mixing in the heavy and light neutrino sectors in models with seesaw.

For the purposes of the present work, we shall distinguish four cases:

(**a**) hierarchical ν_L and degenerate ν_R:

$$
\begin{aligned}
&m_1 \approx 0, \quad m_2 \approx \sqrt{\Delta m_{12}^2}, \quad m_3 \approx \sqrt{\Delta m_{23}^2} \\
&M_{N_1} = M_{N_2} = M_{N_3} \ (\equiv M_N) = M_R \qquad (22)
\end{aligned}
$$

(**b**) (quasi)degenerate ν_L and degenerate ν_R:

$$
\begin{aligned}
&m_1 \ , \quad m_2 \approx m_1 + \frac{1}{2m_1} \Delta m_{12}^2, \\
&m_3 \approx m_1 + \frac{1}{2m_1} \Delta m_{23}^2 \\
&M_{N_1} = M_{N_2} = M_{N_3} \ (\equiv M_N) = M_R \qquad (23)
\end{aligned}
$$

(**c**) hierarchical ν_L and nondegenerate ν_R:

$$
\begin{aligned}
&m_1 \approx 0, \quad m_2 \approx \sqrt{\Delta m_{12}^2}, \quad m_3 \approx \sqrt{\Delta m_{23}^2} \\
&M_{N_1} : M_{N_2} : M_{N_3} = \epsilon_N^3 : \epsilon_N^2 : 1 \qquad (24)
\end{aligned}
$$

(**d**) (quasi)degenerate ν_L and nondegenerate ν_R:

$$
\begin{aligned}
&m_1 \ , \quad m_2 \approx m_1 + \frac{1}{2m_1} \Delta m_{12}^2, \\
&m_3 \approx m_1 + \frac{1}{2m_1} \Delta m_{23}^2 \\
&M_{N_1} : M_{N_2} : M_{N_3} = \epsilon_N^3 : \epsilon_N^2 : 1 \qquad (25)
\end{aligned}
$$

with ϵ_N the hierarchy parameter in the heavy neutrino sector. Taking $\epsilon_N < 1 (> 1)$ deals with both hierarchical (and inverse hierarchical) ordering of heavy neutrinos.

Effects of light neutrino masses and mixing on the right-handed Majorana neutrino sector have been analyzed previously [26]. Relationships between light and heavy neutrino masses and mixings have been obtained by inverting the relationships for the seesaw mechanism. In the limit of non-zero $(U_L)_{e3}$, and assuming hierarchical Dirac neutrino masses, the heavy Majorana neutrino masses are either hierarchical or degenerate. The authors of [26] show that degenerate right-handed neutrino masses correspond to maximal heavy neutrino mixing, while hierarchical ones correspond to heavy neutrino masses which scale linearly with ratios of Dirac neutrino masses. For degenerate right-handed neutrinos, the pertinent product of Yukawa couplings does not depend on the mixing matrix elements or Majorana phases in the heavy neutrino sector. These are cases in which one could test for the parameters in the light neutrino sector. The situation is changed when the right-handed neutrinos are nondegenerate. In the experimentally favored limit of small θ_{13} and the large-mixing angle solution (LMA) of the MSW, the right-handed masses are related to the light neutrino masses and mixing angles through:

$$M_{N_1} \approx \frac{(m_1^D)^2}{m_2}\frac{1}{\sin^2\theta_{12}} \; ; \; M_{N_2} \approx 2\frac{(m_2^D)^2}{m_3} \; ;$$
$$M_{N_3} \approx \frac{1}{2}\frac{(m_1^D)^2}{m_2}\sin^2\theta_{12} \tag{26}$$

and the mixing angles for heavy neutrinos are related to the mixing angles for light neutrinos by:

$$\beta_{12} \approx -\frac{1}{\sqrt{2}}\frac{m_1^D}{m_2^D}\cot^2\theta_{12} \; ; \; \beta_{13} \approx \sqrt{2}\frac{m_1^D}{m_3^D}\cot^2\theta_{12} \; ;$$
$$\beta_{23} \approx -\frac{m_2^D}{m_3^D} \tag{27}$$

where $m_i, i = 1, \ldots 3$ are the light neutrino masses and $m_i^D, i = 1, \ldots 3$ are the Dirac masses.

The product of Yukawa couplings appearing in the renormalization group equations for the slepton mass matrix can then be approximated by [27]:

(a) $$(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij} \approx \frac{M_N^2}{4v_R^2}\delta_{ij} + \frac{M_N}{v^2\sin^2\beta}\sqrt{\Delta m_{23}^2} \times \left(\sqrt{\frac{\Delta m_{23}^2}{\Delta m_{12}^2}}(U_L)_{i2}^*(U_L)_{j2} + (U_L)_{i3}^*(U_L)_{j3}\right)$$

(b) $$(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij} \approx \frac{M_N^2}{4v_R^2}\delta_{ij} + \frac{M_N}{v^2\sin^2\beta}\left[m_1\delta_{ij} + \frac{\Delta m_{23}^2}{2m_1}\left(\frac{\Delta m_{23}^2}{\Delta m_{12}^2}(U_L)_{i2}^*(U_L)_{j2} + (U_L)_{i3}^*(U_L)_{j3}\right)\right]$$

(c) $$(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij} \approx \frac{M_{N_3}^2}{4v_R^2}\left((U_R)_{i1}^*(U_R)_{j1}\frac{M_{N_1}^2}{M_{N_3}^2} + (U_R)_{i2}^*(U_R)_{j2}\frac{M_{N_2}^2}{M_{N_3}^2} + (U_R)_{i3}^*(U_R)_{j3}\right) + \frac{M_{N_3}}{v^2\sin^2\beta}\sqrt{\Delta m_{23}^2} \times \left(\sqrt{\frac{\Delta m_{23}^2}{\Delta m_{12}^2}}(U_L)_{i2}^*(U_L)_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}^*(U_L)_{j3}\right)$$

(d) $$(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij} \approx \frac{M_{N_3}^2}{4v_R^2}\left((U_R)_{i1}^*(U_R)_{j1}\frac{M_{N_1}^2}{M_{N_3}^2} + (U_R)_{i2}^*(U_R)_{j2}\frac{M_{N_2}^2}{M_{N_3}^2} + (U_R)_{i3}^*(U_R)_{j3}\right) + \frac{M_{N_3}}{v^2\sin^2\beta} \times \left[m_1\delta_{ij}\frac{M_{N_1}}{M_{N_3}} + \frac{\Delta m_{23}^2}{2m_1}\left(\frac{\Delta m_{23}^2}{\Delta m_{12}^2}(U_L)_{i2}^*(U_L)_{j2}\frac{M_{N_2}}{M_{N_3}} + (U_L)_{i3}^*(U_L)_{j3}\right)\right] \tag{28}$$

with $v^2 = \kappa_1^2 + \kappa_2^2$, $\tan\beta = \kappa_1/\kappa_2$, and where the ratios of the right-handed neutrino masses and mixing elements can be approximated as in the previous equations.

4 CP violating observables in the leptonic sector

We analyze the effects of phases in the slepton mixing in LFV processes in charged lepton decays and in lepton-flavor conserving dipole moments. Lepton flavor violation is induced by the off-diagonal components in the slepton mass matrix $(\delta\tilde{m}_L^2)$, $(\delta\tilde{m}_R^2)$, and the trilinear slepton mass couplings $\mathbf{A}_L$ and $\mathbf{A}_{LR}$, while leptonic dipole moments are determined by the diagonal components of the same parameters.

4.1 Lepton flavor violation observables

The amplitude for the lepton-flavor violating dipole operator $l' \to l\gamma$ has the form:

$$\mathcal{M}_{l'l\gamma} = \frac{ie}{2m_{l'}}\bar{u}_l(p_2)\sigma^{\mu\nu}q_\nu(a_{Ll'l\gamma}P_L + a_{Rl'l\gamma}P_R)u_{l'}(p_1) + h.c., \tag{29}$$

which leads to the branching ratio:

$$\Gamma_{l'\to l\gamma} = \frac{1}{16\pi^2}(|a_{Ll'l\gamma}|^2 + |a_{Rl'l\gamma}|^2)m_{l'}^5 \tag{30}$$

Processes that violate charged lepton flavor, such as $l' \to l\gamma$ or $\mu Ti(Al) \to eTi(Al)$ can provide important complementary information on the leptonic CP violating phases, even if these processes are CP conserving, because of the extreme precision expected in the measurement of the branching ratios. In particular, beams of low-energy muons more intense by several orders of magnitude than present beams will probe LFV process with high sensitivity $B.R.(\mu \to e\gamma) \sim 10^{-14}$, $B.R.(\mu \to eee) \sim 10^{-16}$. The latter sensitivity opens the way to measuring the T-odd, CP-violating asymmetry $A_T(\mu^+ \to e^+e^+e^-)$.

The supersymmetric contributions to the lepton flavor violation amplitude arise at one-loop level from graphs with chargino-sneutrinos, or neutralino-sleptons, or (specific to LRSUSY) doubly-charged higgsino-sleptons in the loop. It has been shown previously [28] that chargino-sneutrino graphs give the dominant contribution for $(U_L)_{e3} \leq 10^{-4}$. Full expressions for the dipole amplitudes have appeared elsewhere [29]. These expressions enter the evaluation of the T-odd asymmetry, discussed next.

4.2 T-odd asymmetry in $\mu^+ \to e^+e^+e^-$

It has been suggested that the the distribution in the Dalitz plot asymmetry in $\mu^+ \to e^+e^+e^-$ carries information on chirality and the Lorenz structure of LFV couplings absent in $\mu \to e\gamma$ [30]. In particular, the T-odd asymmetry is a sensitive probe of CP-violation in LFV decays. It was shown in MSSM(RN) that this asymmetry is strongly anti-correlated with the $\mu \to e\gamma$ branching ratio, so that it could provide information complementary to the radiative muon decay [31].

The effective Lagrangian for $\mu^+ \to e^+e^+e^-$ is [30]:

$$\begin{aligned}\mathcal{L}_{\mu eee} = -\frac{4G_F}{\sqrt{2}}\Big\{ & m_\mu a_{L\mu e\gamma}\bar{\mu}_L\sigma^{\mu\nu}e_R F_{\mu\nu} \\ & +m_\mu a_{R\mu e\gamma}\bar{\mu}_R\sigma^{\mu\nu}e_L F_{\mu\nu} + g_1(\bar{\mu}_R e_L)(\bar{e}_R e_L) \\ & +g_2(\bar{\mu}_L e_R)(\bar{e}_L e_R) + g_3(\bar{\mu}_R\gamma^\mu e_R)(\bar{e}_R\gamma_\mu e_R) \\ & +g_4(\bar{\mu}_L\gamma^\mu e_L)(\bar{e}_L\gamma_\mu e_L) \\ & +g_5(\bar{\mu}_R\gamma^\mu e_R)(\bar{e}_L\gamma_\mu e_L) \\ & +g_6(\bar{\mu}_L\gamma^\mu e_L)(\bar{e}_R\gamma_\mu e_R) + h.c.\Big\} \end{aligned} \quad (31)$$

Here G_F is the Fermi constant, $a_{(L)R\mu e\gamma}$ the photon penguin amplitude from $\mu \to e\gamma$, g_1, g_2 scalar-type, and g_i, $i = 3, \ldots 6$ vector-type four-fermion coupling constants dependent on the model. Kinematics of the $\mu^+ \to e^+e^+e^-$ is determined by two energies (of the decay positrons) and two angular variables (measuring the direction of muon polarization with respect to the decay plane). Defining the polarization vector with respect to a plane in which the z-axis is the direction of the electron momentum, and the x-axis the direction of the most energetic positron momentum, and assuming 100% polarized muons, the T-odd asymmetry in the three-body decay is:

$$\begin{aligned} A_T &= \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)} \\ &= \frac{3e}{2B.R.(\delta = 0.02)}\Big\{2.0\ \mathrm{Im}(a_{L\mu e\gamma}g_3^* + a_{R\mu e\gamma}g_4^*) \\ &\quad -1.6\ \mathrm{Im}(a_{L\mu\gamma}g_5^* + a_{R\mu e\gamma}g_6^*)\Big\} \end{aligned} \quad (32)$$

where $N(P_y)$ denotes the number of events with positive or negative y-component of muon polarization and $B.R.(\delta = 0.02)$ represents the three body branching ratio, $B.R.(\mu \to 3e)$, maximized for the T-odd asymmetry [30]. Full expressions for all three-bodies LFV decays in LRSUSY have been obtained in [32] and we shall use them in our numerical explorations. Similar results can be obtained for $\tau \to lll$ decays; given the lower experimental precision in the τ versus μ branching ratios, we do not explore $\tau \to 3l$ decays and asymmetries here.

4.3 Lepton flavor conserving observables

The amplitude for the dipole operator responsible for the anomalous magnetic moment of the muon is:

$$\mathcal{M}_\mu = \frac{ie}{2m_\mu}\bar{u}_\mu(p_2)(a_{L\mu}P_L + a_{R\mu}P_R)\sigma^{\mu\nu}q_\nu u_\mu(p_1)A_\mu \quad (33)$$

and the corresponding amplitude for dipole operator responsible for the electron electric dipole moment is:

$$\mathcal{D}_e = -\frac{i}{2}d_e\bar{u}_e(p_2)\sigma_{\mu\nu}F^{\mu\nu}\gamma_5 u_e(p_1) \quad (34)$$

The new measurement for the muon anomalous magnetic moment a_μ [33]:

$$a_\mu^{exp} - a_\mu^{SM} = (2.6 \pm 1.6) \times 10^{-9} \quad (35)$$

gives a 1.6 σ deviation between theory and experiment [34]. The contributions to the anomalous magnetic moment of the muon are related to the amplitudes for the LFV decay $\mu \to e\gamma$, except that they are flavor diagonal. In LRSUSY, as in MSSM with singlet right-handed neutrinos, the Yukawa coupling constants appear in the renormalization group equations in Hermitean combination, thus inducing CP-violating phases in the off-diagonal elements of the slepton masses only. This suppresses the CP violating effects in processes dominated by diagonal slepton mass elements, such as magnetic dipole or electric dipole moments. We use anomalous magnetic moment constraints to restrict the parameters of the model and the electric dipole moments to extract information about the heavy neutrino mixing.

The contributions to the electric dipole moment of the electron come from the same graphs as the muon anomalous magnetic moment, except that both the incoming and outgoing muon have to be replaced by electrons. Since a non-vanishing d_e in the SM results in fermion chirality flip, both CP violation and $SU(2)_L$ symmetry breaking are required. The corresponding contribution will depend on the phases of the model, which are not universal, even if the soft-breaking masses are assumed to be universal. There are several CP-violating phases allowed in the left-right supersymmetric model [35]. Some appear in the gaugino masses for the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$: $M_i = |M_i|exp(i\omega_i)$. Some appear in the soft-symmetry breaking Lagrangian, in the trilinear soft breaking parameter $A_0 = |A_0|exp(i\alpha_0)$, in the quadratic soft-breaking $B_0 = |B_0|exp(i\theta_{B_0})$, and in the Higgs mixing parameter $\mu_{ij0} = |\mu_{ij0}|exp(i\theta_{\mu_{ij}})$. As in MSSM, one can always set one of the gaugino phases to zero. Since we would like to compare the results obtained in LRSUSY with the ones obtained in MSSM, we chose a minimal set of non-zero phases to coincide to mSUGRA phases [36]. The Yukawa couplings appear through diagonal terms in the slepton mixing matrices, therefore the lepton electric dipole moments do not provide information on the CP-violating phases in the neutrino mass matrix. However, in the case of nondegenerate right-handed neutrinos, they give information on the mass ratios of the heavy neutrinos and therefore provide complimentary information on the neutrino sector from LFV decays. It is for this purpose that we include them in this analysis.

We eliminate all other phases in favor of two which we choose to be $(\theta_{ij})_\mu \equiv \delta_{ij}\theta_\mu$ and ω_1. The contributions to the electron dipole moment arise then from the chargino-sneutrino, the neutralino-slepton and the doubly-charged higgsino-slepton graphs. We assume as before the dipole to be saturated by the contributions from diagrams with sleptons in the loop (coming from neutralino diagrams where chirality is flipped internally). For a complete list of contributions, see [35].

5 Numerical analysis

The flavor violating decays and the electric dipole moments are sensitive to the universal GUT parameters m_0 (the scalar mass), the trilinear coupling A_0, the value and the sign of the Higgs mixing parameter μ, the value of $\tan\beta$, and the values of the $U(1)$, left- and right-handed gaugino masses M_1, M_L and M_R (through $m_{1/2}$). We fix the sign of μ to be positive: although $\mu < 0$ can satisfy both the anomalous magnetic moment of the muon and $b \to s\gamma$ constraints, the range is limited [37], while a larger range is permitted for $\mu > 0$. We choose a light supersymmetric particle scenario and set the masses and $\tan\beta$ in a range consistent with muon magnetic moment constraints. We concentrate on contributions with sleptons in the graphs. The diagrams with doubly-charged higgsinos give a contribution smaller by $10^{-2} - 10^{-1}$ than the diagrams with neutralinos.

In numerical evaluations, we fix the gauge and Yukawa couplings at M_Z then use the RGE's up to the scale M_{W_R}, where we introduce the heavy neutrinos, fix their masses, and the light neutrino masses and mixing. We assume universal soft-symmetry breaking at the GUT scale $M_{GUT} \sim 2 \cdot 10^{16}$ GeV. We then run all Yukawa coupling matrices from M_{W_R} to M_{GUT} using the renormalization group equations for LRSUSY [23]. We assume universality of the soft-supersymmetry breaking terms, then run all parameters back to M_{W_R}, where the heavy neutrinos and sneutrinos decouple and LRSUSY breaks to MSSM. We do not consider any intermediate scales between M_{W_R} and M_Z. We then obtain all parameters by running the RGE's to M_Z. We use previously given expressions [32] for chargino and neutralino masses and mixings, then calculate the amplitude for $\mu \to e\gamma$, the T-odd asymmetry in the decay $\mu^+ \to e^+e^+e^-$ with polarized muons, and electron EDM for the scenarios (**a**)-(**d**).

For the light neutrinos masses and mixings, we fix the parameters using the LMA-MSW solution as [38]:

$$\tan^2\theta_{12} = 0.36 \quad \tan^2\theta_{23} = 1.4 \quad \text{and} \quad \tan^2\theta_{13} = 0.005 \tag{36}$$

and, correspondingly, the median values for Δm^2_{atm} and Δm^2_{sol}:

$$\begin{aligned} \Delta m^2_{atm} &\approx \Delta m^2_{23} = 3 \times 10^{-3}\ \mathrm{eV}^2 \\ \Delta m^2_{sol} &\approx \Delta m^2_{12} = 3 \times 10^{-5}\ eV^2 \end{aligned} \tag{37}$$

This sets the neutrino mass parameter in the light sector $\epsilon_\nu = \sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{23}}} = 0.1$. In the heavy neutrino sector, we set $M_{N_3} = 5 \times 10^{14}$ GeV and explore the leptonic branching ratios and EDM's for a variety of values of the parameter $\epsilon_N = \sqrt{\frac{M_{N_2}}{M_{N_3}}}$. In order to obtain reasonable values for the triplet coupling $\mathbf{Y}_{LR}$, we choose $v_R = 3.2 \times 10^{14}$ GeV. This insures that the largest Yukawa coupling of the triplet Δ_R^{--} is within experimental bounds, $(Y_{LR})_{\tau\tau} \le 0.8$. The drawback of such heavy scales is that the W_R boson is superheavy, $M_{W_R} \approx 1.5 \times 10^{11}$ TeV.

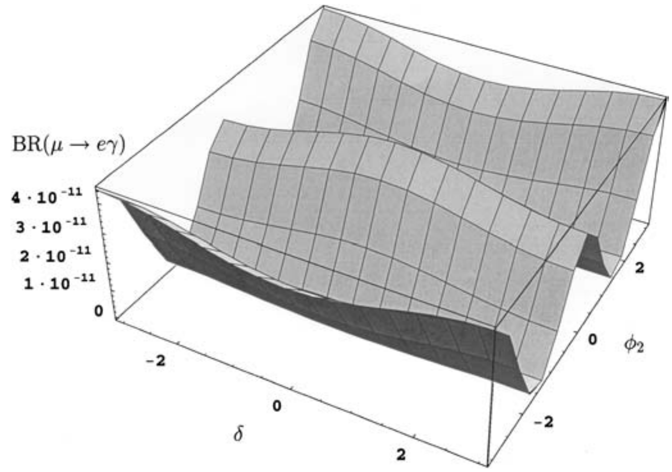


Fig. 1. The dependence of the branching ratio $B.R.(\mu^+ \to e^+\gamma)$ on the CP violating angles in the light neutrino sector, ϕ_2 and δ in scenario (**a**). The values of the other parameters were chosen for all representative figures as follows: $M_L = 100$ GeV, $m_0 = 200$ GeV, $\mu = 200$ GeV, $a_0 = m_0$ and $\tan\beta = 10$

Before begining our detailed numerical analysis, we make some general explanatory comments. In all scenarios, the angles θ_{12}, θ_{23}, θ_{13} from the light neutrino mixing matrix, and β_{12}, β_{23}, β_{13} from the heavy neutrino mixing matrix are set by solar and atmospheric neutrino constraints and the inverse seesaw relationships (26). Scenarios (**a**) and (**b**), involving degenerate heavy neutrinos, do not probe the heavy neutrino mass spectrum or mixing but constitute a probe into the CP phases of the light neutrino spectrum. By contrast, scenarios (**c**) and (**d**) depend on the masses, mixings and CP phases in the heavy neutrino spectrum. In some cases this dependence dominates; in others it is obscured by interference from the light neutrino mixings and phases. Lepton EDM's do not depend on the CP phases in the neutrino mass matrix, but for scenarios (**c**) and (**d**), they depend on the mass ratios in the heavy neutrino sector. We analyze these observables in scenarios (**a**)-(**d**) in detail below.

In both scenarios (**a**) and (**b**), the strongest dependence of the LFV branching ratio and the T-odd asymmetry is on the angles ϕ_2 and δ. Here, as in MSSM(RN), there is an anti-correlation of asymmetry and branching ratio results [31]. This can be seen by comparing Fig. 1, where we show the variation of $B.R.(\mu \to e\gamma)$ with ϕ_2 and δ, with Fig. 2, where we show the CP-odd asymmetry $A_T(\mu^+ \to e^+e^+e^-)$ with the same parameters. Unlike the branching ratio, which is more sensitive to variations in the CKM-angle δ, the asymmetry is a sensitive probe of the angle ϕ_2 from the light neutrino sector. We analyze this dependence in more detail. In scenario (**a**) the asymmetry, as a function of the CKM angle δ, varies by a factor of 4 in the light sparticles scenario for different Majorana

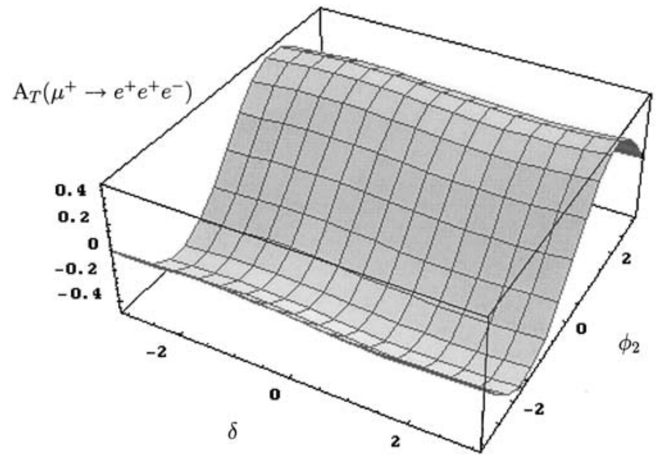


Fig. 2. The dependence of the T-odd asymmetry $A_T(\mu^+ \to e^+e^+e^-)$ on the CP violating angles in the light neutrino sector, ϕ_2 and δ in scenario (**a**). The values of the other parameters were chosen as in Fig. 1

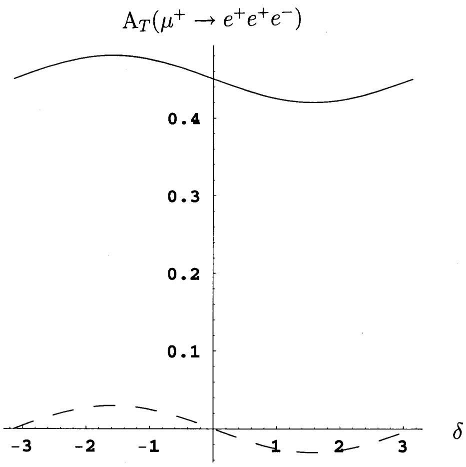


Fig. 3. The dependence of the T-odd asymmetry $A_T(\mu^+ \to e^+e^+e^-)$ on the CKM-like angle in the light neutrino sector δ in scenario (**a**), for $\phi_2 = \frac{\pi}{2}$ (solid curve) and $\phi_2 = 0$ (dashed curve). We take the other LRSUSY parameters as in Fig. 1

angles ϕ_2 (Fig. 3). The T-odd asymmetry is 0 for $\phi_2 = 0$ and maximal for $\phi_2 = \frac{\pi}{2}$. The dependence on the angle ϕ_2, (Fig. 4), shows no significant differences between $\delta = 0$ and $\delta = \frac{\pi}{2}$ scenarios. The same features apply to scenario (**b**). There are no clear distinguishing signs in LRSUSY between scenarios with hierarchical (or inverse hierarchical) light neutrinos (**a**), and degenerate (or quasidegenerate) light neutrinos (**b**), within models with degenerate right-handed neutrinos. Although there are slight differences in the absolute values of the T-odd asymmetry and branching ratios, these differences are small and the dependence on the angles very similar. Such small differences can easily be absorbed in a change of mass parameters, so we conclude that CP phases dependence cannot distinguish between scenarios (**a**) and (**b**).

The situation is more involved when we allow for non-degenerate heavy neutrinos.

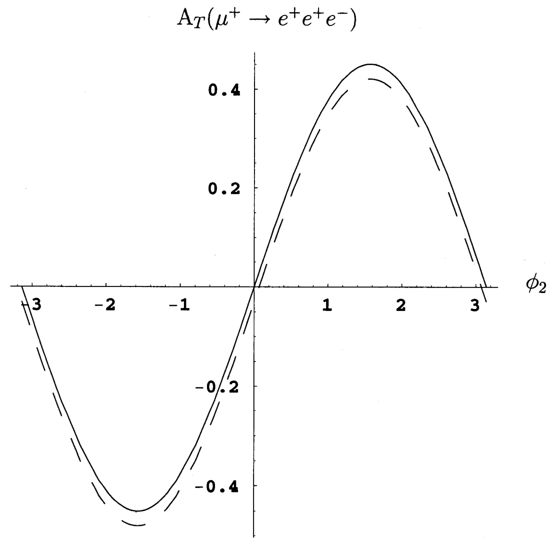
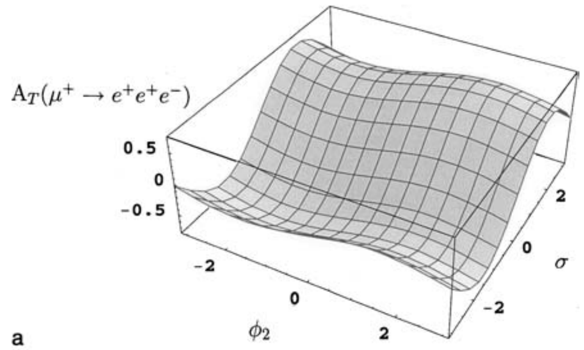


Fig. 4. The dependence of the T-odd asymmetry $A_T(\mu^+ \to e^+e^+e^-)$ on the Majorana angle in the light neutrino sector ϕ_2 in scenario (**a**), for $\delta = 0$ (solid curve) and $\delta = \frac{\pi}{2}$ (dashed curve). We take the other LRSUSY parameters as in Fig. 1

In Fig. 5a we plot the T-odd asymmetry in the three-body polarized muon decay as a function of ϕ_2 and σ, in scenario (**c**) and Fig. 5b the same quantity, again as a function of ϕ_2 and σ, in scenario (**d**), for $\epsilon_N^2 = 0.1$. The asymmetry behaves quite differently in the two cases (hierarchical heavy neutrinos with either hierarchical or degenerate light neutrinos). The T-odd asymmetry can be used to distinguish between the last two scenarios, (**c**) and (**d**). The anti-correlation of the T-odd asymmetry for scenarios (**c**) and (**d**) with the branching ratio of $\mu \to e\gamma$ persists. If we compare the branching ratio for $\mu^+ \to e^+\gamma$ and for $\mu^+ \to e^+e^+e^-$, they show the same dependence on the CP-violating phases, comfirming the dominance of the photon penguin in both decays. Improved precision measurements of both would favor the dipole decay as more restrictive of the parameter space. However, because of the anti-correlation between the branching ratio and the corresponding T-odd asymmetry in $\mu^+ \to e^+e^+e^-$, measurements of both $B.R.(\mu^+ \to e^+\gamma)$ (or $B.R.(\mu^+ \to e^+e^+e^-)$) and $A_T(\mu^+ \to e^+e^+e^-)$ could provide information on CP violating phases. Note that the asymmetry in LRSUSY can be as large as 4%, larger than estimates in $SO(10)$, but smaller than possible values in $SU(5)$ [30]. The largest $B.R.(\mu^+ \to e^+e^+e^-)$ is obtained for negative values of $A_T(\mu^+ \to e^+e^+e^-)$ and could reach 10^{-14}, which could be measured by experiments with a sensitivity at 10^{-16} level.

Finally, we analyze leptonic electric dipole moments within LRSUSY with leptonic CP phases. It is a well-known problem in MSSM, persisting in some versions of LRSUSY [35], that for light superparticle masses and large minimal supergravity (mSUGRA) phases, the predicted EDM for the electron is 2-3 orders of magnitude larger than the experimental bound [39,40]. Three remedies exist to this problem: (i) assuming a heavy slepton spec-



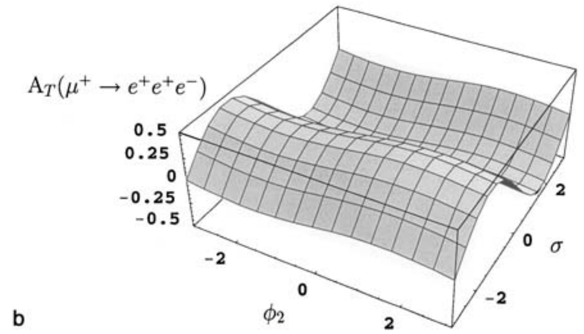


Fig. 5a,b. (a) The T-odd asymmetry in the three-body polarized muon decay $A_T(\mu^+ \to e^+e^+e^-)$ as a function of ϕ_2 and σ in scenario (**c**); and **(b)** the same quantity, again as a function of ϕ_2 and σ in scenario (**d**). We chose $\epsilon_N^2 = 0.1$ for both curves and the rest of the parameters as in Fig. 1

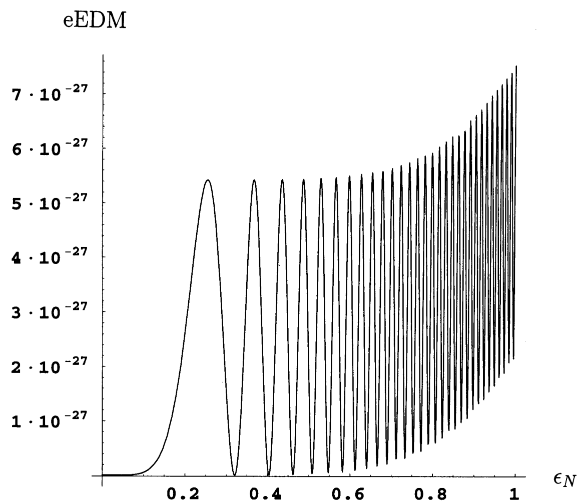


Fig. 6. The dependence of the electron EDM on the heavy neutrino mass parameter ϵ_N in scenario (**d**). We take θ_μ, $\omega_1 \approx 5 \times 10^{-2}$ and the other LRSUSY parameters as in Fig. 1

trum, with masses in the several TeV region, and large phases [39]; (ii) assuming light supersymmetric particles and phases of order 10^{-3} [40]; or (iii) assuming cancellations between chargino and neutralino contributions to the EDM [41]. Since our set of parameters involves light masses, and we analyze the neutralino contribution only, we work in scenario (ii) and assume the mSUGRA phases to be small. As discussed previously, the EDM's are independent of the CP phases in the neutrino sector since phases appear only in the off-diagonal matrix elements $(Y_L^\dagger Y_L + Y_{LR}^\dagger Y_{LR})_{ij}$. However the electric dipole moments are a sensitive measure of the mass textures in the heavy neutrino sector. Since the heavy neutrino textures are hierarchical, the renormalization group effects do not interfere with their generic structure. The effect on leptonic EDM's is very different from MSSM(RN), since the CP-conserving phases in the heavy neutrino mass matrices depend on the hierarchy parameter ϵ_N. Figure 6 shows the dependence of the electron EDM on ϵ_N in scenario (**d**). The rapid oscillation of EDM's with ϵ_N is a typical feature of LRSUSY, and always occurs in the case of non-degenerate, hierarchical or anti-hierarchical, heavy neutrinos. It does not depend on the details of neutrino mass ordering in the light sector. As a general feature, the oscillatory behaviour is more rapid for increasing values of ϵ_N. For some values of the parameter ϵ_N, the electron EDM can reach and exceed the experimental bounds, and for the entire parameter space considered, it is within reach of future experiments. This oscillatory dependence raises the interesting possibility that the EDM is very small (i.e., within experimental bounds) in LRSUSY due to the right-handed neutrino mass structure. The same oscillatory behaviour occurs in LFV decays, but there it is obscured by dependence on several other phases and more difficult to isolate.

6 Conclusion

We have presented an analysis of the effects of Majorana and CKM-like phases in the heavy and light neutrino sectors in a fully left-right supersymmetric model. In this model, lepton flavor violation is introduced by radiative corrections in the slepton mass matrix. Because of left-right symmetry and the Higgs structure of the model (chosen to support the seesaw mechanism), the LFV parameters depend on the mixing elements in both the heavy and light neutrino sectors. We express the heavy neutrino mixings as functions of the light neutrino parameters within present constraints from atmospheric and solar neutrinos within the LMA of the MSW. We use this parametrization to study heavy neutrino mass textures and CP violating phases effects in the T-odd asymmetry in polarized muon three-body decay $\mu^+ \to e^+e^+e^-$ and the electric dipole moment of the electron. We look at four cases, assuming either degenerate or non-degenerate heavy neutrinos, and either (quasi)degenerate or hierarchical light neutrinos. The possibility of inverse hierarchy of the heavy neutrinos is included as well. Although our phenomenological analysis is general but not exhaustive, it is illustrative of the general features of the model.

With respect to the dependence on the CP violating angles in the light neutrino sector, we found that, as in MSSM(RN) the information obtained from the LFV branching ratios and T-odd asymmetry are anti-correlated. However, the similarity ends here, as the dependence of $B.R.(\mu \to e\gamma)$ and $A_T(\mu^+ \to e^+e^+e^-)$ is different in LRSUSY from MSSM(RN), even in the case in which the heavy neutrinos are degenerate. More importantly, in the case of non-degenerate right-handed neutrinos, the LFV can provide information on the CP-violating Majorana and CKM-like phases in the heavy neutrino sector. This is one new feature of the model.

If one analyzes the dependence of the LFV processes or EDM's on the hierarchical mass parameter in the heavy neutrino sector, they all exhibit an oscillatory dependence, rather than the smooth curves seen in MSSM(RN). This is the second new, distinguishing feature of the model. It persists independent of the textures in the light neutrino sector, and it becomes more enhanced with increasing ϵ_N, the mass hierarchy parameter in the heavy sector.

As general features, scenarios with degenerate right-handed neutrinos are useful for testing CP-violating phase dependence in the light sector. For the Majorana phase ϕ_2, the T-odd asymmetry provides the most promising signals. For the CKM phase δ, stronger tests would come from improved precision in measuring the branching ratios of either $\mu \to e\gamma$ or $\tau \to \mu\gamma$. Scenarios with hierarchical right-handed neutrinos are most promising testing grounds for CP violating phases in the heavy neutrino spectrum. The T-odd asymmetry appears most sensitive to the CKM phase σ, while the branching ratio for $\tau \to \mu\gamma$ is a more sensitive observable with respect to the Majorana angle ψ_3; neither is sensitive to the angle ϕ_2 in these scenarios.

In conclusion, the dependence on the light and heavy neutrino masses, mixings and CP-violating phases in the LRSUSY model exhibits very different features from the MSSM with right handed singlet neutrinos. It can be used to test the heavy neutrino sector masses and mixings, but also to serve as a distinguishing signal of left-right symmetry within the realm of supersymmetric models.

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